Game Theory

Themes

1. Introduction to Game Theory
2. Sequential Games
3. Simultaneous Games
4. Conclusion

Introduction to Game Theory

Game theory is the branch of decision theory concerned with interdependent decisions. The problems of interest involve multiple participants, each of whom has individual objectives related to a common system or shared resources. Because game theory arose from the analysis of competitive scenarios, the problems are called *games* and the participants are called *players*. But these techniques apply to more than just sport, and are not even limited to competitive situations. In short, game theory deals with any problem in which each player's strategy depends on what the other players do.

Situations involving interdependent decisions arise frequently, in all walks of life. A few examples in which game theory could come in handy include:

- Friends choosing where to go have dinner
- Parents trying to get children to behave
- Commuters deciding how to go to work
- Businesses competing in a market
- Diplomats negotiating a treaty
- Gamblers betting in a card game

All of these situations call for strategic thinking – making use of available information to devise the best plan to achieve one's objectives. Perhaps you are already familiar with assessing costs and benefits in order to make informed decisions between several options. Game theory simply extends this concept to interdependent decisions, in which the options being evaluated are functions of the players' choices.

The appropriate techniques for analyzing interdependent decisions differ significantly from those for individual decisions. To begin with, despite the rubric *game*, the object is not to "win." Even for strictly competitive games, the goal is simply to identify one's optimal strategy. This may sound like a euphemism, but it is actually an important distinction. Using this methodology, whether or not we end up ahead of another player will be of no consequence; our only concern will be whether we have used our optimal strategy. The reasons for this will become clear as we continue.

In gaming, players' actions are referred to as *moves*. The role of analysis is to identify the sequence of moves that you should use. A sequence of moves is called a *strategy*, so an optimal strategy is a sequence of moves that results in your best outcome. (It doesn't have to be unique; more than one strategy could result in outcomes that had equal payoffs, and they would all be optimal, as long as no other strategy could result in a higher payoff.)

There are two fundamental types of games: sequential and simultaneous. In sequential games, the players must alternate moves; in simultaneous games, the players can act at the same time. These types are distinguished because they require different analytical approaches. The sections below present techniques for analyzing sequential and simultaneous games, and we conclude with a few words about some advanced game theory concepts.

Sequential Games

To analyze a sequential game, first construct a *game tree* mapping out all of the possibilities.[*] Then follow the basic strategic rule: "**look ahead and reason back**:" [†]

1. Look ahead to the very last decision, and assume that if it comes to that point, the deciding player will choose his/her optimal outcome (the highest payoff, or otherwise most desirable result).
2. Back up to the second-to-last decision, and assume the next player would choose his/her best outcome, treating the following decision as fixed (because we have already decided what that player will pick if it should come to that).
3. Continue reasoning back in this way until all decisions have been fixed.

[*] For those familiar with decision trees, game trees are quite similar. The main difference is that decision trees map decisions for one person only, while game trees map decisions for all players.

[†] All rules and most examples here have been borrowed from: Dixit, Avinash K., and Barry J. Nalebuff. Thinking Strategically. New York: W. W. Norton & Co., 1991. This is an excellent nontechnical book on game theory, and this paper (intended for educational use only) is heavily based upon it.

That's all there is to it. If you actually play out the game after conducting your analysis, you simply make the choices you identified at each of your decisions. The only time you even have to think is if another player makes a "mistake." Then you must look ahead and reason back again, to see if your optimal strategy has changed.

Notice that this procedure assumes that the other players are as smart as we are, and are doing the same analysis. While this may not be the case, it is the only safe assumption. If it is correct, we will have made our best possible decision. For it to be incorrect, an opponent must choose an option not in his/her own best interests.

The analytical process is best illustrated through an example. Suppose that a company called Fastcleaners currently dominates the market and makes $300,000 per year, and we are considering starting a competing company. If we enter the market, Fastcleaners will have two choices: accept the competition or fight a price war. Suppose that we have done market analyses from which we expect that if Fastcleaners accepts the competition, each firm will make a profit of $100,000 (the total is less than Fastcleaners alone used to make because they could enjoy monopoly pricing). However, if Fastcleaners fights a price war, they will suffer a net loss of $100,000, and we will lose $200,000. (Note that these are the ultimate payoffs, not just temporary gains or losses that may change over time.)

With this information, we can build a game tree (Figure 3.4.1). We begin by mapping the decision structure before including any data: we ("us") move first, and either enter the market or do not. If we enter, Fastcleaners ("FC") gets to respond, and either accepts us or starts a price war. If we do not enter, nothing happens. Then we just fill in the numbers listed above, and the tree is complete.

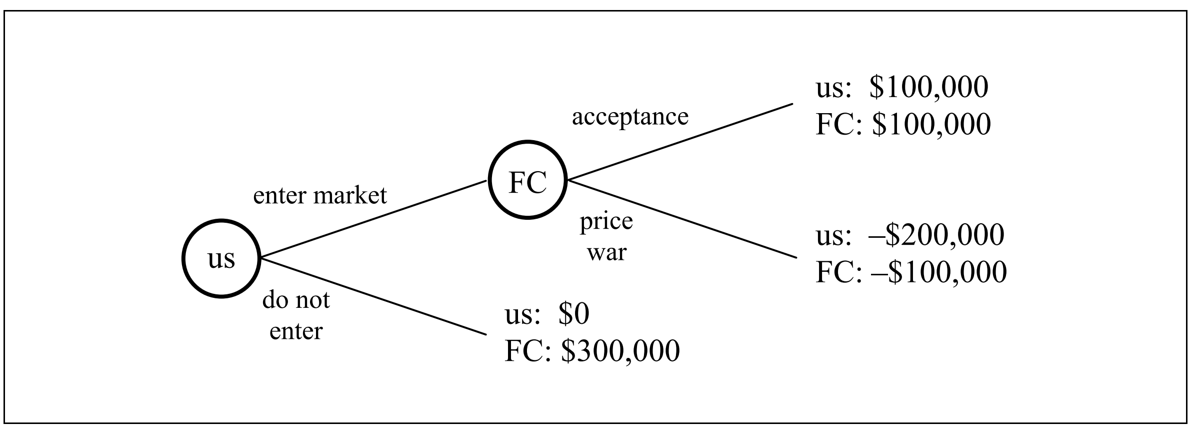

Figure 3.4.1: Cleaners Example Game Tree

Now we can look ahead and reason back. Looking ahead, if Fastcleaners faces the last choice, it will be between $100,000 profit and $100,000 loss. Naturally, they will choose the profit. Reasoning back, we now know it will not come to a price war, which means our decision is between $100,000 profit and $0 profit. Consequently, we decide to start our company and enter the market, where we expect to make $100,000 profit.

Of course this is only a very simple example. A more realistic situation might involve more decision stages (Fastcleaners could begin a price war, and re-evaluate every month) or more factors (Fastcleaners could be a chain, willing to accept a loss at this branch in order to build a reputation of toughness to deter other would-be competitors), but the analytical method of looking ahead and reasoning back will remain valid. It has been proven that there exists an optimal strategy for any sequential game involving a finite number of steps. Note that this doesn't always mean it is possible to determine. The game of chess technically has an optimal strategy, but no one has yet been able to map out all of the possible combinations of moves. Only specific scenarios have been solved.

We end this section with a few observations before moving on to simultaneous games. First, notice that looking ahead and reasoning back determines not just one player's optimal strategy, but those for all players. It is called the *solution* to the game. Once it has been determined, it is irrelevant whether or not the game is actually played, as no one can possibly do better than the solution dictates.[*] That is why the concept of "winning" does not really apply. Alternatively, one could argue that the player who gets to make the last decision wins. Sequential games are determined, so ultimately, there are only two choices: either the player with the last decision gets his/her best outcome, or the game is not played. Thus, the game tree obviates the need to actually play out the game.

Simultaneous Games

Turning to simultaneous games, it is immediately apparent that they must be handled differently, because there is not necessarily any last move. Consider a simple, but very famous example, called the Prisoner's Dilemma: two suspected felons are caught by the police, and interrogated in separate rooms. They are each told the following:

- If you both confess, you will each go to jail for 10 years.
- If only one of you confesses, he gets only 1 year and the other gets 25 years.
- If neither of you confesses, you each get 3 years in jail.

[*] The only exception is if someone makes a mistake, and moves differently than planned in his/her strategy. Note that this is very definitely an error; it cannot possibly result in a better outcome for the player – or it would have been part of his/her optimal strategy – and it almost always results in a worse one.

We cannot look ahead and reason back, since neither decision is made first. We just have to consider all possible combinations. This is most easily represented with a *game table* listing the players' possible moves and outcomes. Table 3.4.1, below, presents the outcomes for the first prisoner, for each possible combination of decisions that he and the other prisoner could make:

Table 3.4.1: Prisoner's Dilemma Game Table

		First Prisoner's Decision	
		Confess	*Hold Out*
Other Prisoner's Decision	*Confess*	10 years	25 years
	Hold Out	1 year	3 years

The game table (also called a payoff matrix) clearly indicates if that the other prisoner confesses, the first prisoner will either get 10 years if he confesses or 25 if he doesn't. So if the other prisoner confesses, the first would also prefer to confess. If the other prisoner holds out, the first prisoner will get 1 year if he confesses or 3 if he doesn't, so again he would prefer to confess. And the other prisoner's reasoning would be identical.

There are several notable features in this game. First of all, both players have *dominant strategies*. A dominant strategy has payoffs such that, regardless of the choices of other players, no other strategy would result in a higher payoff. This greatly simplifies decisions: **if you have a dominant strategy, use it**, because there is no way to do better. Thus, as we had already determined, both prisoners should confess. Second, both players also have *dominated strategies*, with payoffs no better than those of at least one other strategy, regardless of the choices of other players. This also simplifies decisions: **dominated strategies should never be used**, since there is at least one other strategy that will never be worse, and could be better (depending on the choices of other players). A final observation here is that if both prisoners use their optimal strategies (confess), they do not reach an optimal outcome. This is an important theme: maximizing individual welfare does not necessarily aggregate to optimal welfare for a group. Consequently, we see the value of communication. If the two prisoners could only communicate, they could cooperate and agree to hold out so they would both get lighter sentences. But without the possibility of communication, neither can risk it, so both end up worse off.

Although it was very simple, the above example laid the groundwork for developing strategies for simultaneous games:

- If you have a dominant strategy, use it.
- Otherwise, look for any dominated strategies and eliminate them.

Many games can be solved using these steps alone. Note that by eliminating a dominated strategy, you cross off a whole row or column of the game table, which changes the remaining strategies. Accordingly, if you can eliminate a dominated strategy, you should immediately check to see if you now have a dominant strategy. If you do not, then look for another dominated strategy (there may have been more than one originally, or you may have just created one or more). You can keep iterating in this way until you either find a dominant strategy, or the game cannot be reduced any further.

For example, consider the news magazines *Time* and *Newsweek*, each trying to choose between cover stories about AIDS and the national budget. The game table below presents information for both players. (Many analysts find this staggered payoff notation, invented by Thomas Schelling, more convenient than a separate table for each player.) In each outcome box, the upper-right value represents *Newsweek*'s payoff, while the lower-left value represents *Time*'s payoff (for enhanced clarity in this example, the *Newsweek* outcomes are colored blue, and the *Time* outcomes are colored red). Thus we see that if *Newsweek* and *Time* both choose AIDS for their cover stories, for example, *Newsweek* will get 28% of the readers, and *Time* will get 42%. (The other 30% of readers are only interested in the budget story, so they would buy neither magazine in that case.)

Table 3.4.2: Time/Newsweek Cover Story Game Table

		***Newsweek* Cover Story**	
		AIDS	*Budget*
***Time* Cover Story**	*AIDS*	28% (Newsweek) 42% (Time)	30% (Newsweek) 70% (Time)
	Budget	70% (Newsweek) 30% (Time)	12% (Newsweek) 18% (Time)

Now we can analyze this table to determine each magazine's optimal strategy. *Time* has a dominant strategy: selecting AIDS for its cover story. This move is dominant because no matter which topic *Newsweek* chooses, *Time* would get a higher percentage of readers by running an AIDS cover story than it would by running a budget cover story. Thus *Time*'s optimal strategy is obvious. For *Newsweek*, however, there are no dominant or dominated strategies; its best choice depends upon *Time*'s decision. However, *Newsweek* can see from the game table that *Time*'s dominant strategy is to choose AIDS, and so knows that will be *Time*'s choice. Given this information, *Newsweek*'s optimal strategy becomes selecting the national budget for its cover story (as this will attract 30% of the readers, while competitively running the AIDS cover story would only attract 28%).

<u>Equilibrium</u>

What happens if the game cannot be reduced and there is no dominant strategy? An example might *Time* and *Newsweek* trying to decide what price to charge to for each magazine. If *Newsweek* picks a fairly high price, *Time* could pick a slightly lower one and get most of the swing readers (people who will buy either magazine, as opposed to loyal readers of a specific one). On the other hand, if *Newsweek* picks a very low price, *Time* would do better to set its price a little higher, foregoing the swing readers to make a profit off of its loyal readers. This price response behavior for *Time* is depicted below.

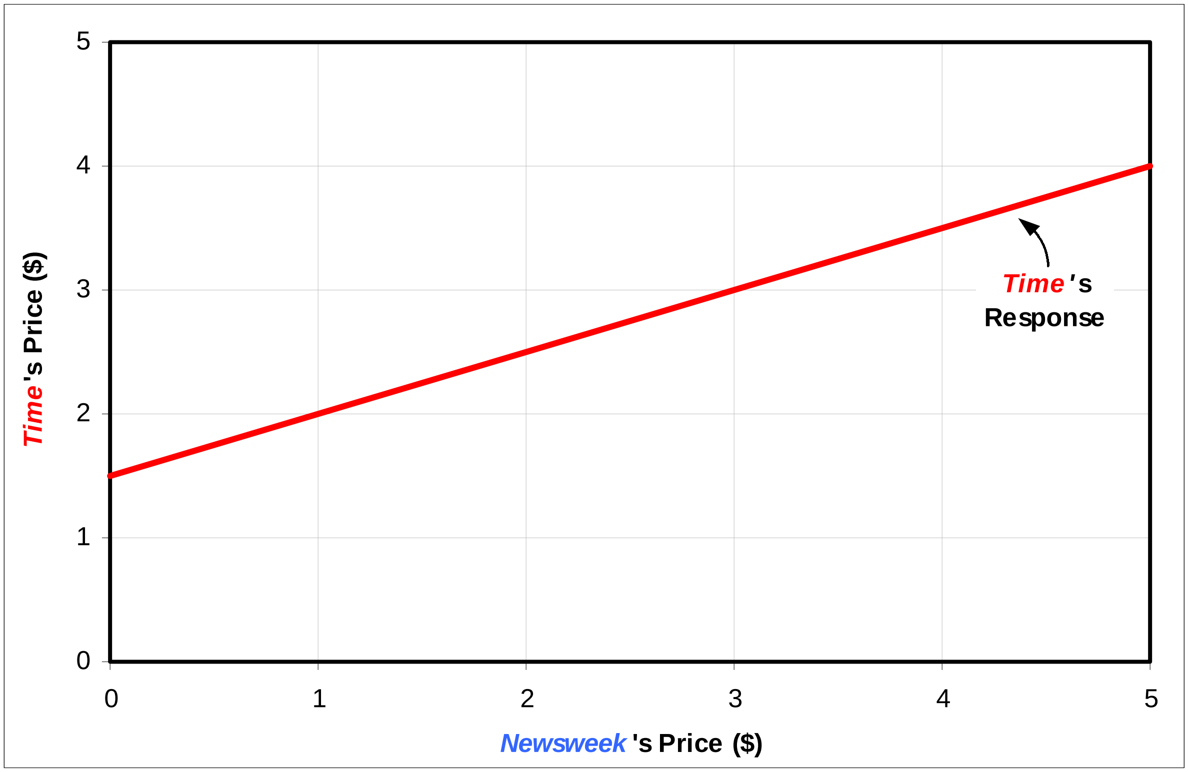

Figure 3.4.2:* Time*'s Price Response to* Newsweek *Prices

However, this is not a sequential game; Time does not have the luxury of waiting for Newsweek to pick a price first. To consider the whole story, let us add Newsweek's best response to Time's price. Figure 3.4.3 presents curves for each magazine's best responses to the other's prices.

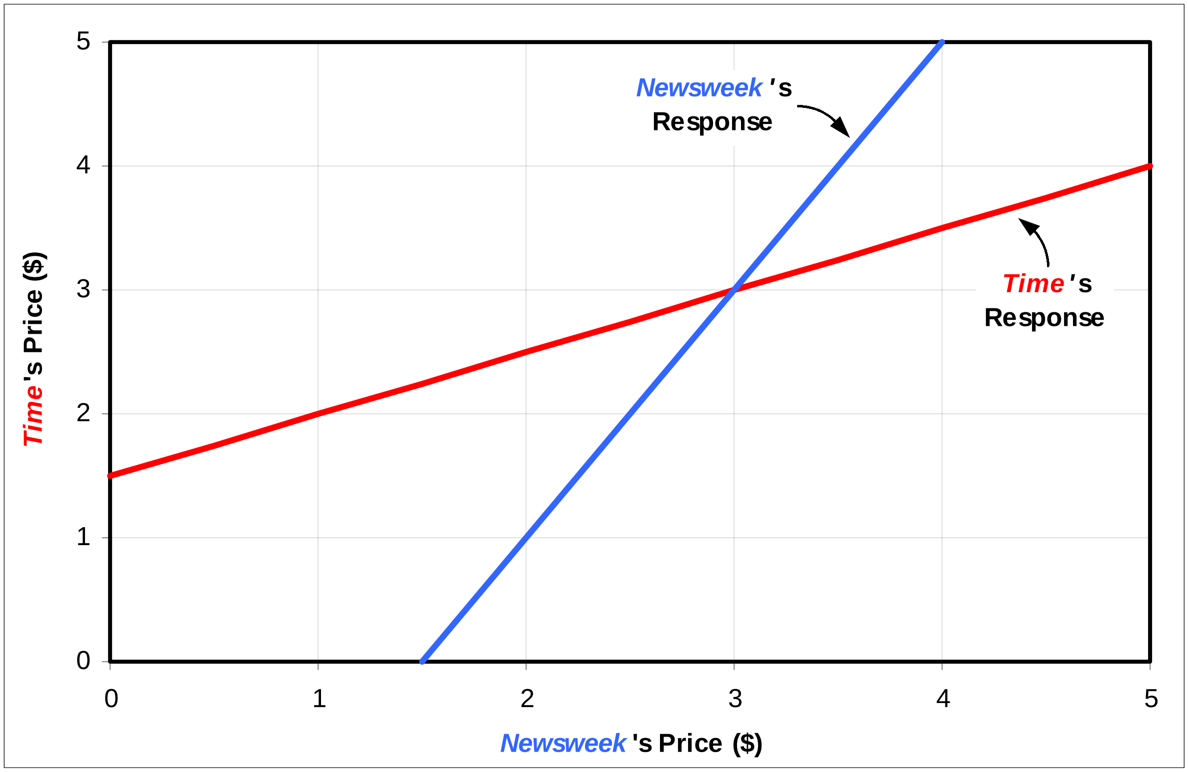

Figure 3.4.3:* Time *and* Newsweek *Price Responses

Strategically, this would appear to produce an endless circle: if *Newsweek* sets its price at \$1, then *Time* should pick \$2, in response to which *Newsweek* would switch to \$2.50, in response to which *Time* would switch again… But there is a way out: **seek equilibrium**. An *equilibrium* (or Nash equilibrium) is a set of outcomes such that no players have any incentive to change strategy.

Notice that the two price response curves intersect. The point at which they cross is an equilibrium – a set of prices such that each magazine is already at its best responses to each other's price. (In this example the prices happen to be \$3 for each, but they need not be equivalent to be an equilibrium.) At this point, neither magazine would have any incentive to raise or lower its price, because to do so would result in a lower profit. Consequently, if there is an equilibrium solution, it represents stability, and is usually the best solution. (Note, however, that a given equilibrium point may not be acceptable to all parties – stability does not necessitate optimality – so compensation or other agreements may be necessary. This is a more advanced aspect of game theory.)

Beyond Equilibrium

There remain two more situations: what if there are multiple equilibrium points, or none? In either case, the optimal choice is a *mixed strategy*, in which players strategically switch between various non-dominated strategies. It is possible to calculate the optimal mixture – the percentage of time each strategy should be used – for any given game, but that is beyond the scope of this discussion. Suffice to conclude by reiterating that if you don't have a dominant strategy, you should seek an equilibrium or mixed strategy.

Conclusion

Game theory is exciting because although the principles are simple, the applications are far-reaching. Interdependent decisions are everywhere, potentially including almost any endeavor in which self-interested agents cooperate and/or compete. Probably the most interesting games involve communication, because so many layers of strategy are possible. Game theory can be used to design credible commitments, threats, or promises, or to assess propositions and statements offered by others. Advanced concepts, such as brinkmanship and inflicting costs, can even be found at the heart of foreign policy and nuclear weapons strategies – some the most important decisions people make.